27 November 1967

Dear Jacquet,

Thanks for your letter. I was a little disappointed that the calculations of my letter to Weil were mostly unnecessary for I rather enjoyed making them. I think that now by making use of two papers of Kirillov in the Доклады I can complete the results of my letter to Weil and give a complete treatment of the non-archimedean case. This will make it possible to give a rather simple formulation of the final result. However I have first to verify the results of Kirillov. As soon as I have done this I will send a revised form of paragraphs 6 and 7 of my letter to Weil. I suppose he will show them to you.

Once the results are in final form we can act on the suggestion of your letter.

Would you please mention to Weil that I have not forgotten that I owe him a reply to his letter.

Yours,

R. Langlands

Ankara

January 22, 1968

Dear Jacquet,

I am sending you at the same time as this letter some notes connected with my previous letter to Weil. The purpose of these notes is to show that if one has a space of automorphic forms on which, roughly speaking, $G_{\mathbb{A}}$ acts irreducibly then to every continuous homomorphism χ of $k^*\backslash I$ into $\mathbb{C}^*$ there is associated a Dirichlet series with an Euler product and a functional equation of the usual type. Moreover for our purposes the representation will be a tensor product $\otimes\tau_{\mathfrak{p}}$ of representations of the local groups and the factor of the Euler product corresponding to a given prime depends only on $\tau_{\mathfrak{p}}$ and the restriction $\chi_{\mathfrak{p}}$ of χ to $k_{\mathfrak{p}}^*$. The factor appearing in the functional equation is also a product of local factors depending only on $\tau_{\mathfrak{p}}, \chi_{\mathfrak{p}}$, and the restriction to $k_{\mathfrak{p}}$ of a given character of $k\backslash\mathbb{A}$. Moreover in general each local representation is associated to a continuous homomorphism $\omega_{\mathfrak{p}}$ of the units of a two dimensional semi-simple algebra over $k_{\mathfrak{p}}$. There is a very simple relation between the factor in the Euler product corresponding to $\tau_{\mathfrak{p}}$ and the local zeta function for $\omega_{\mathfrak{p}}$. The same is true of the factor appearing in the functional equation. The relation is so simple that the converse theorem about the existence of automorphic forms corresponding to a given Euler product is immediately applicable to the Hecke L-series over a quadratic extension of the ground field. You will notice that the carefulness of the exposition declines as the note proceeds (but not I hope to the point of error). You can attribute this to my loss of patience. In particular I give function fields rather short shrift in the last paragraph. It should be a minor matter to take care of them. You may have noticed that, in the previous letter, I made one or two blunders in connection with function fields. There are two unproved lemmas in the notes. These are Lemma 5.2 and the Plancherel Theorem of Gelfand and Graev. The latter, which I want to verify, you probably understand better than I. The situation with the former is explained in the notes. I am looking forward to receiving the notes you promised. I have not yet received anything nor have I heard from Godement. I would have sent him a copy of the notes I am sending you but there are no reproducing facilities here except carbon paper.

Yours,

R. Langlands

March 14, 1968

Dear Jacquet,

I sent you a letter this morning and received your notes this afternoon. Since I will not be writing you again for some time I thought I should acknowledge receipt of them immediately.

By the way your idea of taking the product of two forms is similar to the ideas used by Selberg and Rankin in their discussion of the Ramanujan Conjecture. See for example Selberg's talk at the Pasadena conference on number theory.

I have not yet had a chance to study your notes but it certainly looks as though your proofs are much better than mine. However I think that if one wants, with a view to generalization or arithmetical applications, to understand these particular series well it is best to write out the results as explicitly as possible.

Yours,

R. Langlands

March 14, 1968

Dear Jacquet,

I have decided that it was imprudent and possibly incorrect to assume that the representations described in paragraph 2 of my notes would be the only ones occurring in the Plancherel formula. Even granting the results of Gelfand and Graev there is a possibility, indeed a likelihood, that other representations will be necessary when the characteristic of the residue field is 2. For this reason I ask you to delete Lemma 2.10 from the notes (By the way you should have received these notes long ago; I sent them by air mail more than a month ago).

This deletion does not effect paragraph 3 or 4 but paragraph 5 is incomplete. In other words, for now, we have to assume that the characteristic space of automorphic forms to which we are attaching the Dirichlet series transforms according to a representation $\otimes\tau_{\mathfrak{p}}$ such that, when $\mathfrak{p}$ is a finite prime, $\tau_{\mathfrak{p}}$ is one of the representations constructed in paragraph 2. The converse theorem is also incomplete. In other words there may be Dirichlet series attached to characteristic spaces of automorphic forms which it does not catch.

I am looking into this matter now but it may be a matter of months before I have it completely straightened out. I will keep in touch with you. If you have any information, one way or the other, about the existence of the representations when the characteristic of the residue field is 2, please pass it on to me.

I will be returning to the U.S. during July. If you are still in Princeton I shall try to see you.

Yours,

R. Langlands

Istanbul

21 July 1968

Dear Jacquet,

Thanks for your letter. I'm glad you are coming to Yale for a while.

I was very excited by your theorem. Because of the results of Shimizu and others I was convinced that such a theorem was true but had no idea, in fact still have no idea, how to prove it. I look forward to seeing your proof. I had always thought it would involve the Selberg trace formula. I suppose that in general to every irreducible space of automorphic forms on a twisted form of a given split group corresponds an irreducible space of automorphic forms in the quasi-split group itself. A similar thing will have to be true of the irreducible representations over a local field, archimedean or not. Everything fits together more and more all the time.

I'm on my way back now and look forward to seeing you.

R. Langlands